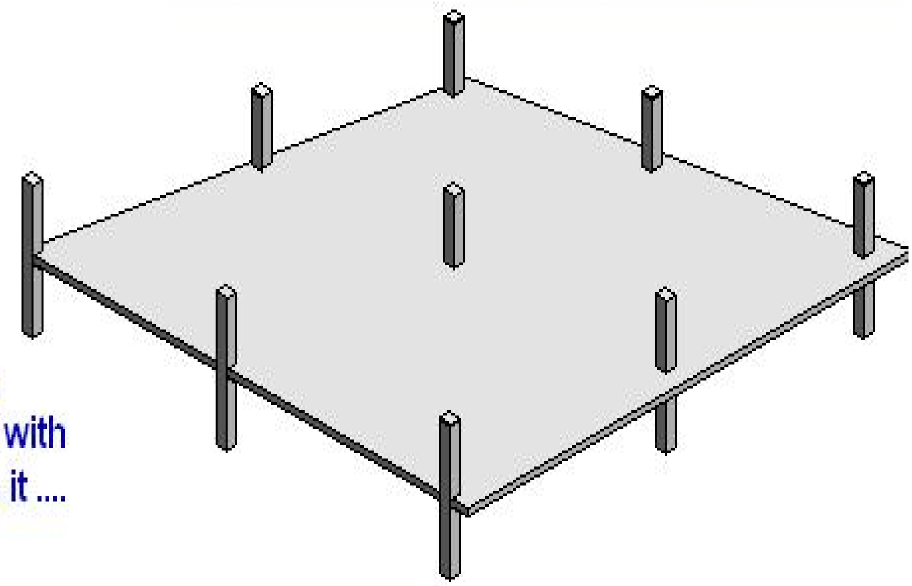
# Flat Slabs



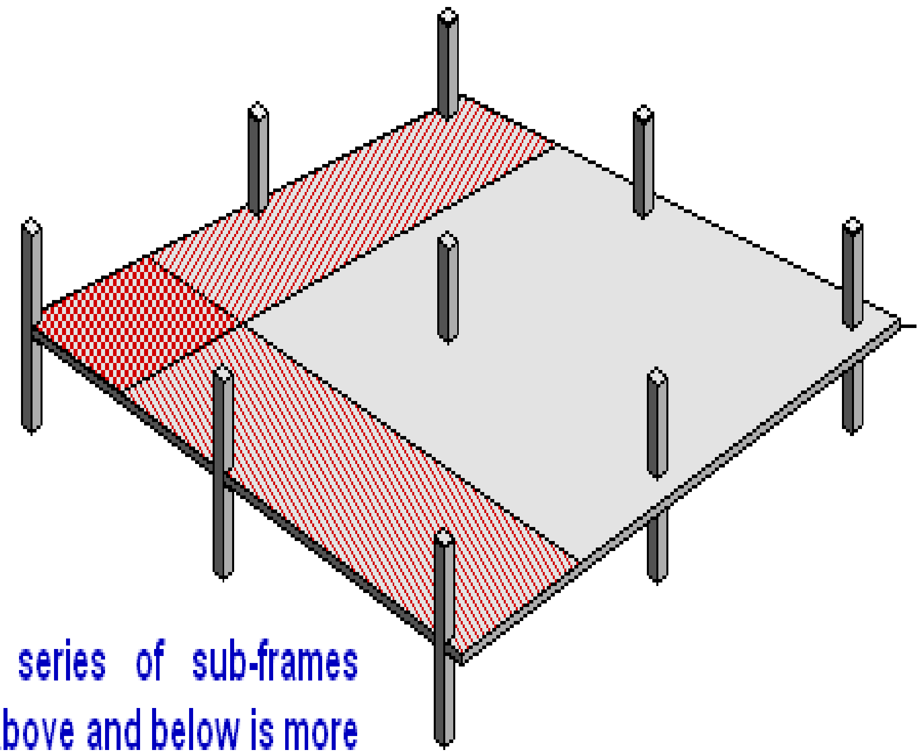
A flat slab floor is a reinforced concrete slab supported directly by the columns with no intermediate beams. Because of this it ....

- is simple to construct, requiring the minimum of formwork,
- minimises construction depths, and
- provides a clear soffit for routing services below the slab.

However, the absence of beams means that the slab has to ....

- carry the shear forces, which are concentrated around the column,
- transmit the moment to the edge and corner columns,
- suffer greater deflections.

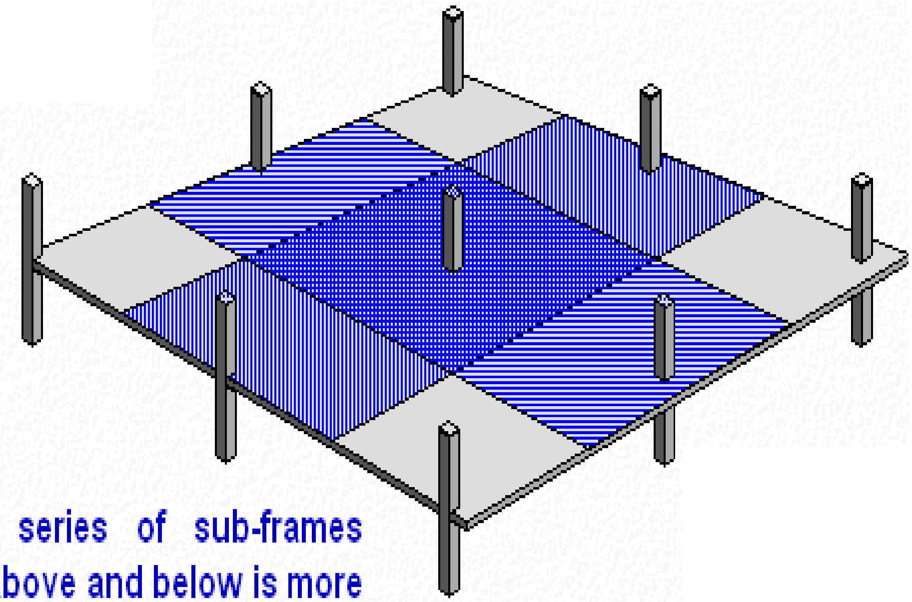
# Flat Slabs - Analysis



The slab can be analysed by the Equivalent Frame method. This provides an acceptable representation of the behaviour of the floor by a system of columns and slab strips analysed separately in each direction.

Rather than a full height frame, a series of sub-frames comprising a single floor with columns above and below is more commonly used (see Analysis of Structures), subject to the most unfavourable arrangements of load. The final moments can be redistributed.

The width of the frame strips is taken between mid-points of columns or the edge of the slab as appropriate. In the slab above the edge strips are :-



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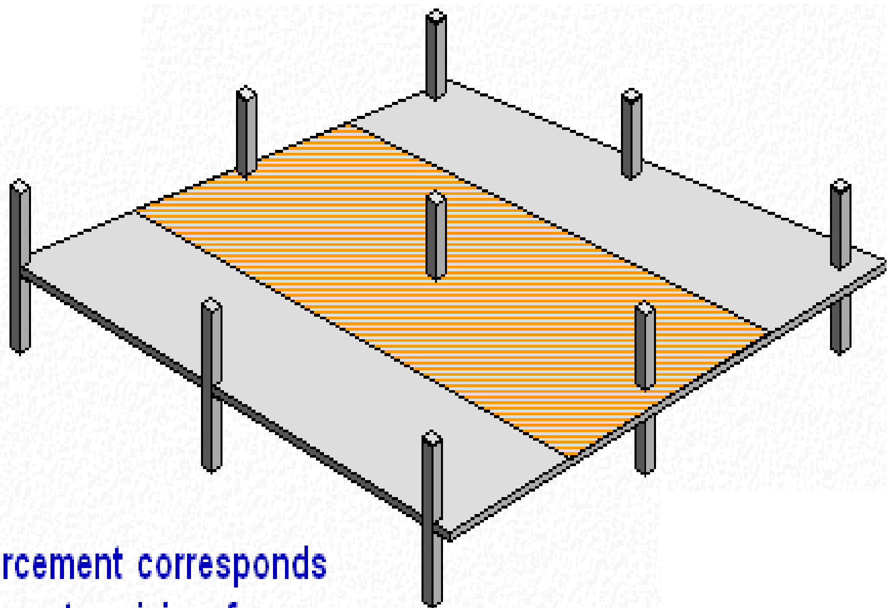
The width of the frame strips is taken between mid-points of columns or the edge of the slab as appropriate. In the slab above the edge strips are :-
and the internal strips are :-

When considering only vertical loads, the stiffness of the slab strip may be based on the full width. In any combination of loading that includes lateral loads the stiffness should be based on half the width.

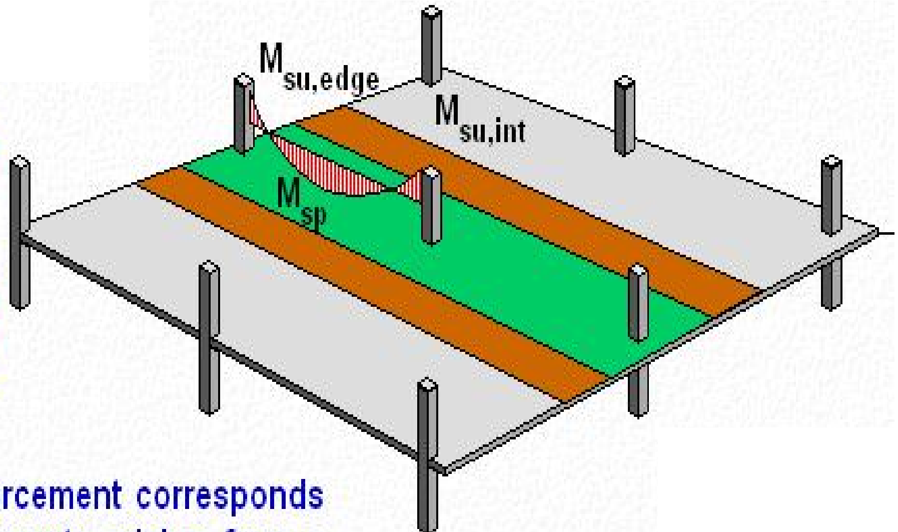
When analysing a frame in one direction the full load should be used because a slab supported just by columns can fail as a one-way mechanism and it should be reinforced to resist the moment from the full load in each direction.



The moments obtained from an analysis of the equivalent frame are the total moments on the slab strip. However, the distribution of moment across the width of the strip is quite obviously not uniform, since the slab is only supported in the centre of the strip.

To ensure that the distribution of reinforcement corresponds approximately to the distribution of moments arising from a rigorous analysis of the slab system, the slab strip is divided into a column strip and a middle strip (half each side), thus:

# Flat Slab - Moments

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To ensure that the distribution of reinforcement corresponds approximately to the distribution of moments arising from a rigorous analysis of the slab system, the slab strip is divided into a column strip and a middle strip (half each side), thus:

and the design moments are apportioned between the two strips as follows :-

| | Column strip | Middle strip |
|---|---|---|
| Negative moment at edge column, $M_{su,edge}$ | 100% | |
| Negative moment at internal column, $M_{su,int}$ | 75% | 25% |
| Positive moment in span, $M_{sp}$ | 55% | 45% |

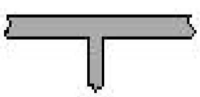
but limited to $0.15 b_e d^2 f_{cu}$

The *widths* of the column and middle strips and the *distribution* of the reinforcement are chosen to reflect the behaviour of the slab.

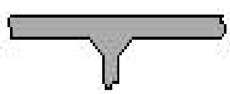
# Widths

In general, the reinforcement required in each column and middle strip should be **distributed uniformly** across the strip.

An exception to this is the reinforcement to resist the negative moment in the column strip at an internal support, $\mathbf{M_{su,int}}$ for a solid slab without drops, thus:

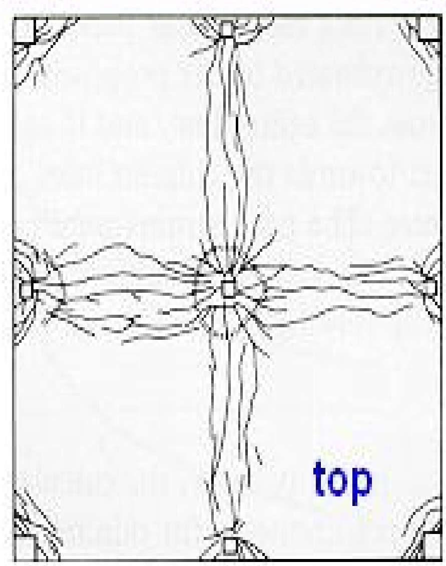


or

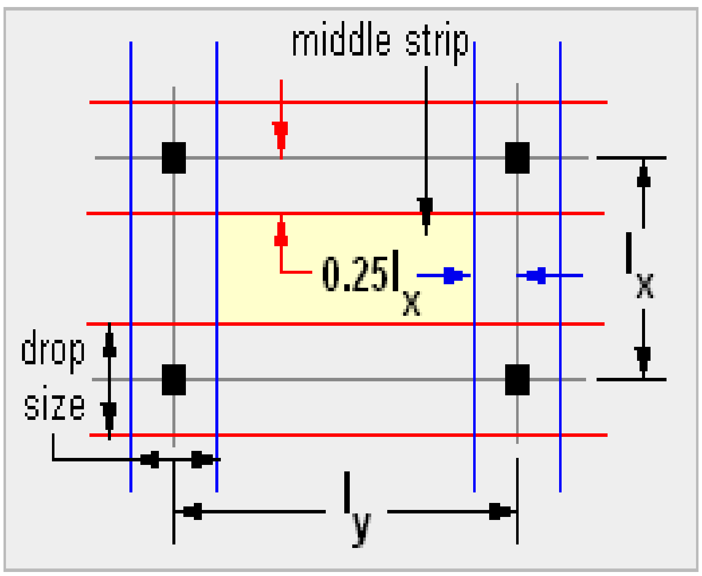


In this case **2/3rds** of the reinforcement is placed in a strip **half** the width of the column strip and central with the column, and the remaining **1/3rd** is placed in the other half of the column strip. This effectively produces **3 strips** as shown.

The final crack patterns in a test of a 4 panel flat plate would seem to justify this approach.



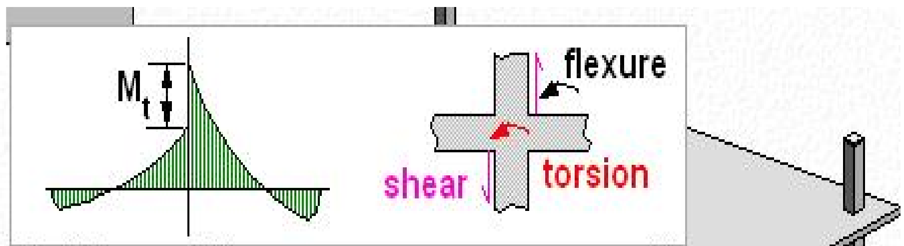
# Distribution



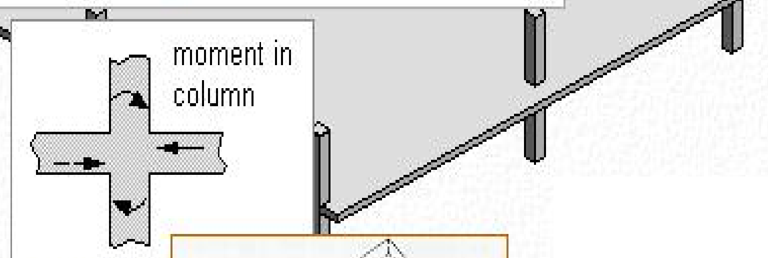

If the slab has drops then the width of the column strip is taken as the drop width, provided this is not less than $l_x / 3$.

The column strip should be symmetrical about an internal column. If adjacent panel dimensions are unequal then the width of the column strip should be based on the larger dimension.

# Flat Slab – Moment Transfer

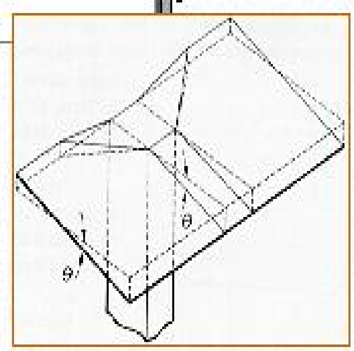



The moment, $M_t$ is transferred from the slab to the column by 3 mechanisms; flexure, torsion and shear.

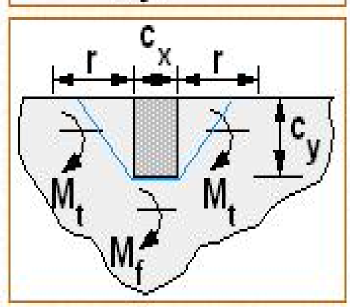



At internal columns the mechanism is aided by the compression couple ('push-push' effect) which the continuous slab exerts, thus:

At edge columns there is no slab continuity and flexural and torsional cracking of the slab close to the column face reduces the moment transfer capacity. A typical yield line pattern (see later pages) will look like this :-

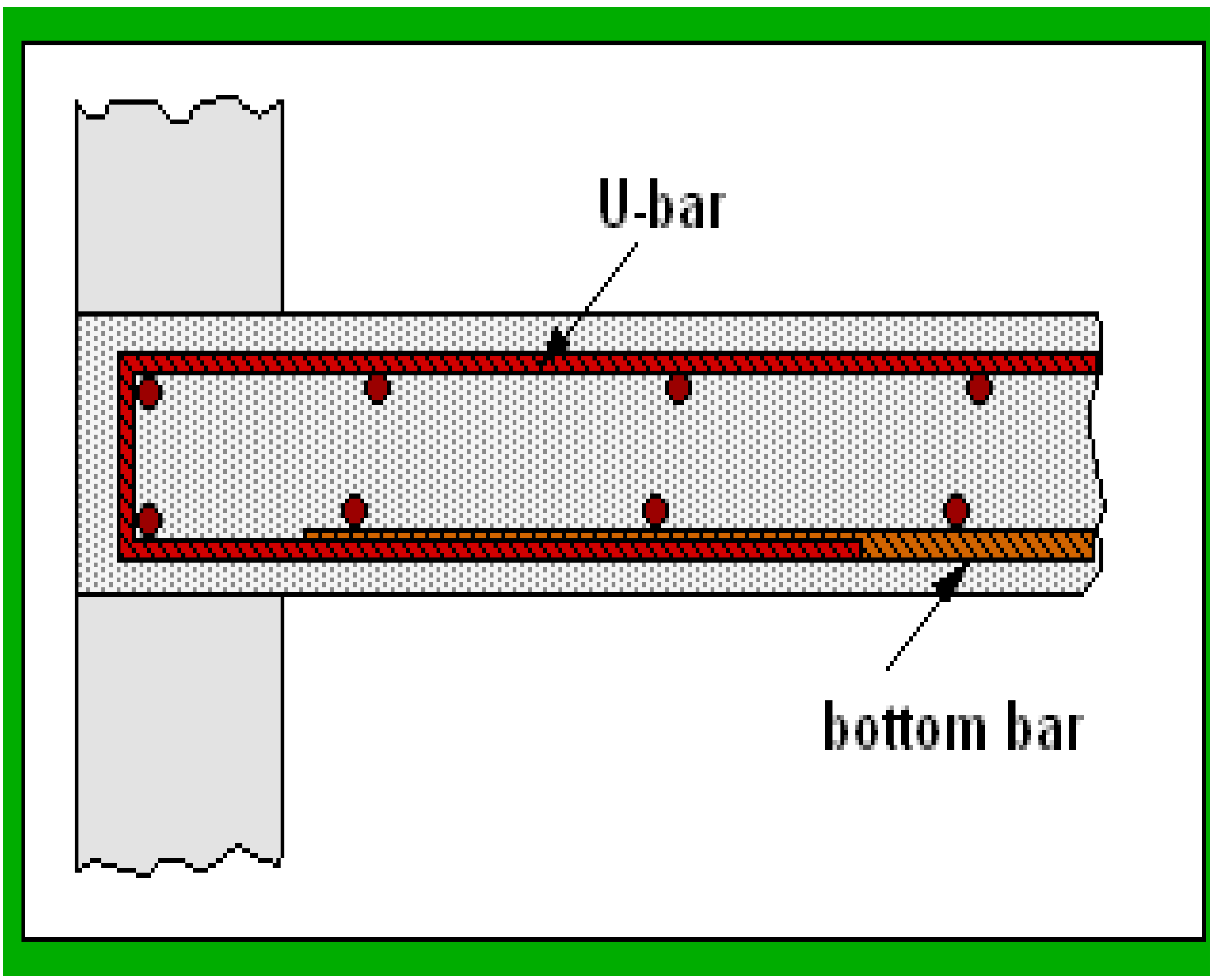



The moment transferred comprises a component $M_f$ resisted by steel passing through the column face, and two components $M_t$ resisted by steel distributed over widths r on either side of the column, which are transmitted to the column as torsions on its side faces.

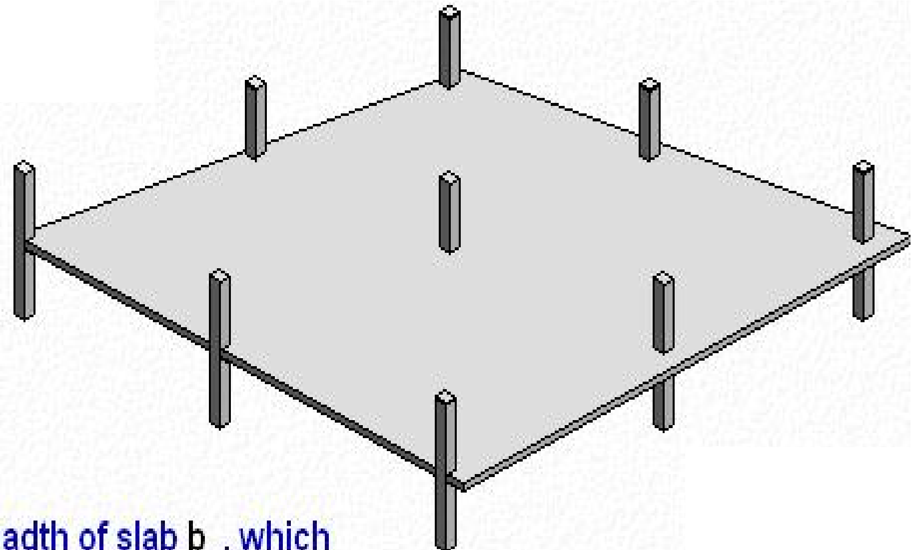



The width of slab over which moment transfer is effective is $c_x + 2r$, where r is limited to $c_y$ for slabs up to 300 mm depth, and to $1.67c_y$ for greater slab depths, provided adequate torsional reinforcement is provided over this length. Generally, U-bars at the edge are satisfactory.

# U-Bars



# Flat Slab – Moment Transfer



A limit must be placed on the moment transferred to an edge column, $M_{t,max}$ to ensure that the slab section is not over-reinforced. This limit is :-

$$M_{t,max} = 0.15\, b_e\, d^2\, f_{cu}$$

The moment is based on an effective breadth of slab $b_e$, which *depends* on the column size and the column position relative to the slab edge.

$M_{t,max}$ is empirical. The relationship assumes a limiting value for the neutral axis depth.

0.6d

0.55d

0.5d

0.45d

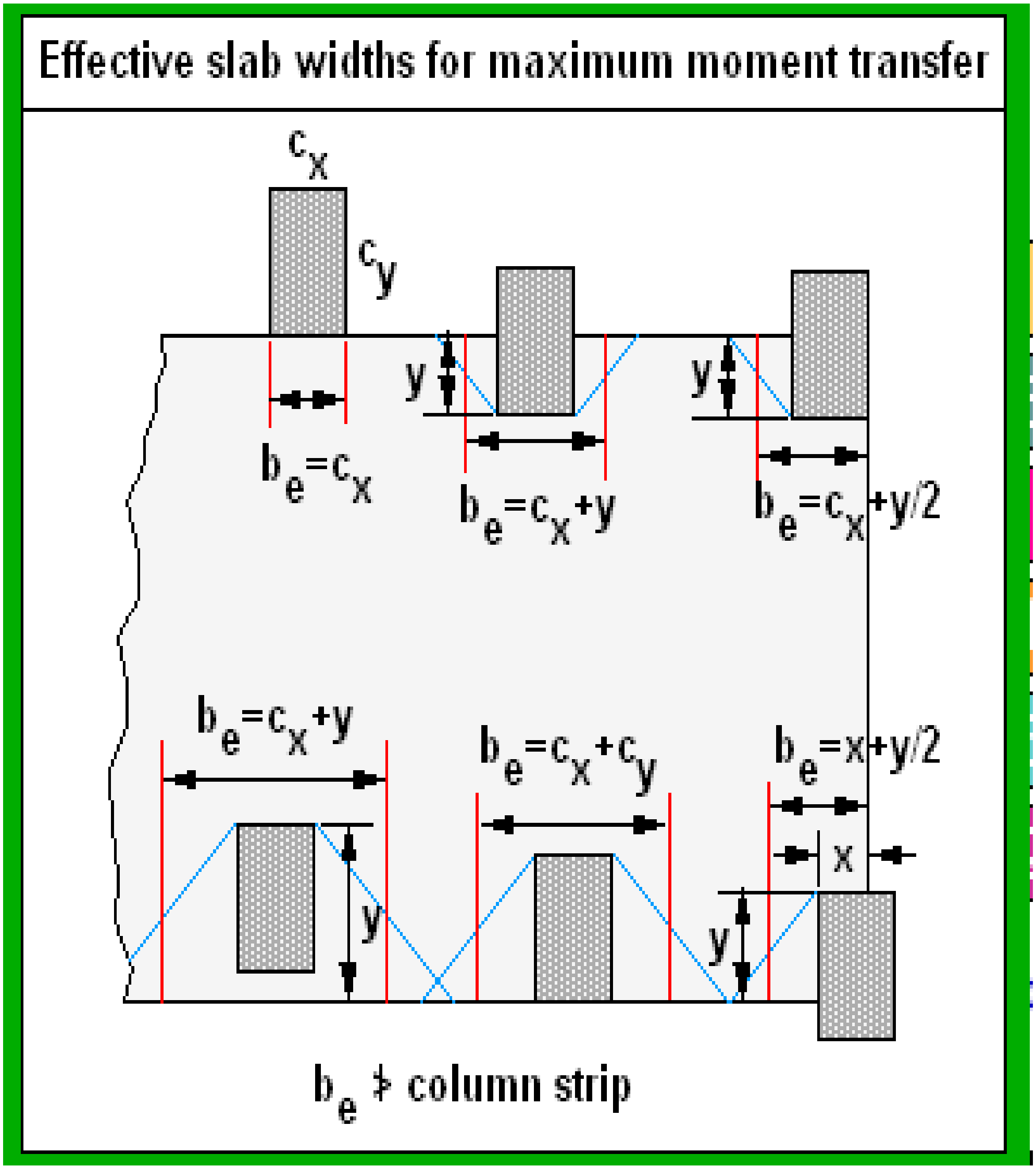
0.35d

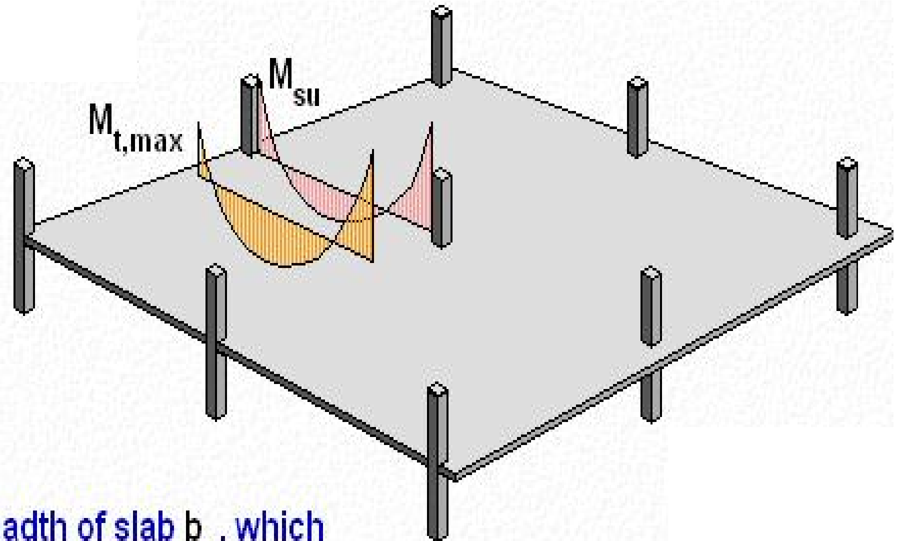
0.25d

Setting x = kd and using a simplified rectangular stress block, then taking moments about the tension steel for a singly reinforced rectangular section gives :

$$M_{t,max} = 0.9kdb_e\,(0.67f_{cu}/1.5)\,(d - 0.45kd) = 0.15\, b_e d^2 f_{cu} \quad \text{giving } k = 0.5$$

# What does it depend on?



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$$M_{t,max} = 0.15\, b_e\, d^2\, f_{cu}$$

The moment is based on an effective breadth of slab $b_e$, which *depends* on the column size and the column position relative to the slab edge.

If the elastic moment from the analysis $M_{su}$ exceeds $M_{t,max}$ then it may be reduced and the positive span moment increased accordingly. If the analysis is by grillage or finite element then the reduction in moment should not exceed 30% . If the equivalent frame method is used then the reduction should not exceed 50%. The difference allows for the conservatism of the latter method.

If the reduced moment still exceeds $M_{t,max}$ then the design should be altered to reduce the moment transfer in one of the following ways :-

- *provide edge beams*
- reduce the span
- reduce the loading
- change column position and add a small cantilever
- increase the strength of the concrete
- increase slab depth

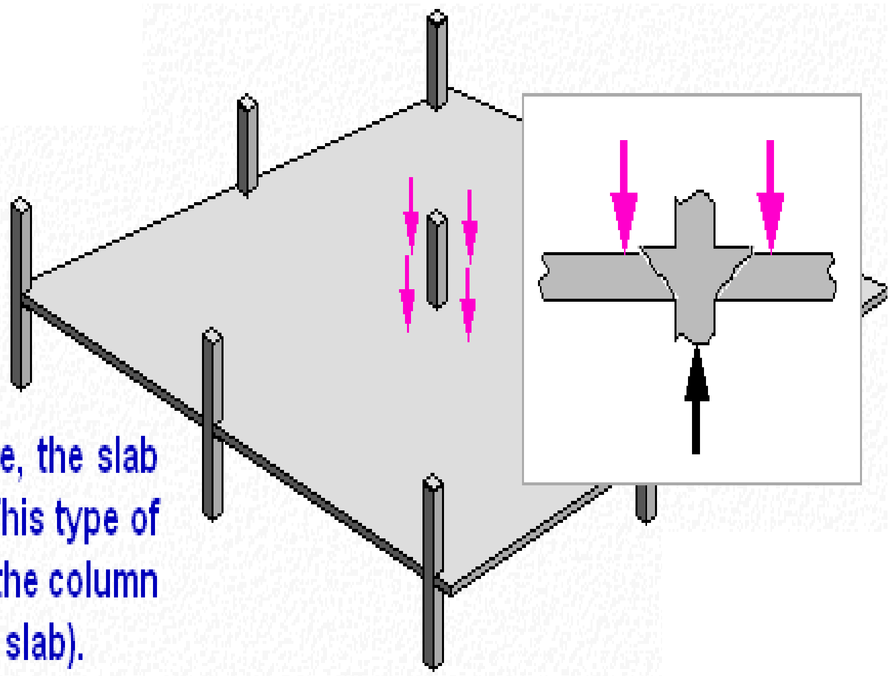
# Edge Beams

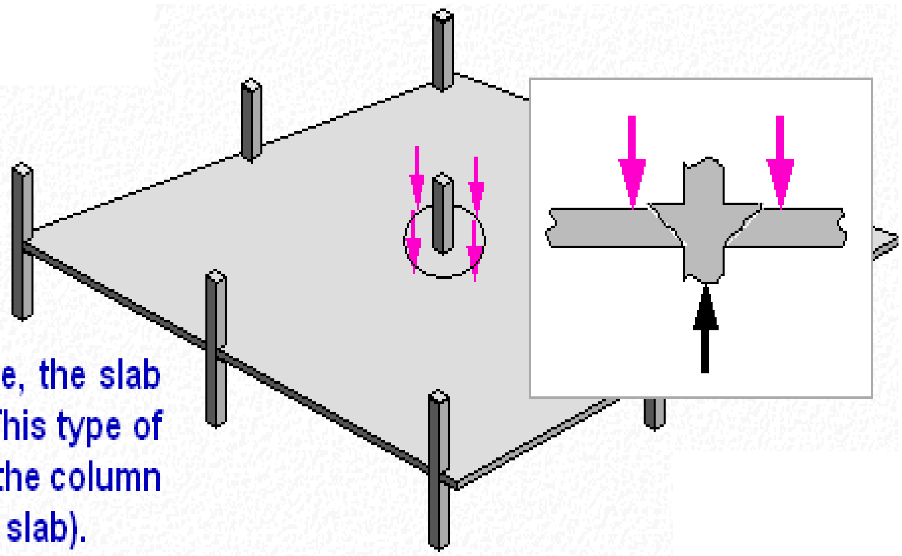
This slab has beams spanning between the edge columns and the internal columns, as well as having edge beams.
This layout does not interfere with the service runs.

# Flat Slab - Shear

With no beams to transfer the load to the columns, there is a concentration of shear force around the column head which has to be resisted by the slab.

If the shear force exceeds this resistance, the slab will be 'pushed down' over the column. This type of failure is called a punching shear failure (the column can be considered to 'punch through' the slab).





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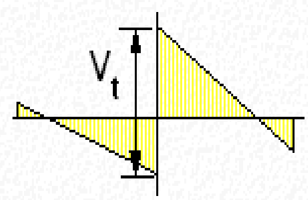
If the shear force exceeds this resistance, the slab will be 'pushed down' over the column. This type of failure is called a punching shear failure (the column can be considered to 'punch through' the slab).

The failure surface of the slab forms a truncated cone or pyramid around the column. The line of failure on the top surface of the slab is called the critical perimeter.

The design is based on the total shear force, $V_t$ at the column face.




If moment is transferred to the column the shear distribution is not uniform and so an *effective shear force*, $V_{eff} = \beta V_t$ is used to take account of local concentrations. For an equivalent frame analysis the direction giving the greater value of $V_t$ is used.

To prevent punching shear failure the slab can be provided with shear reinforcement or its equivalent, the columns provided with splayed heads, or deeper sections of slab, known as drops, provided around the column head. Punching shear is discussed fully in the next topic.

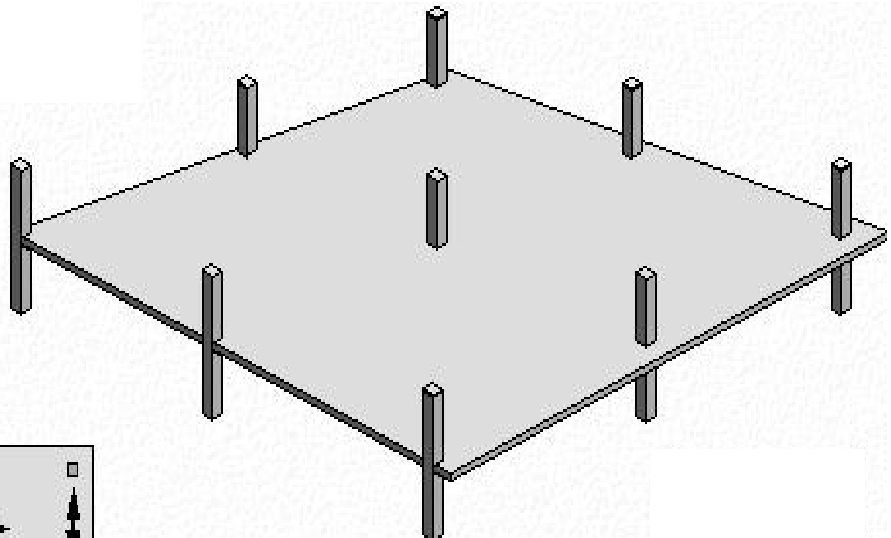
# Effective Shear Force

| Effective shear force for flat slab, $V_{eff} = \beta V_t$ | |
|---|---|
| Condition | $\beta$ |
| Internal column<br>$1 + (1.5M_t / V_t x)$ or | 1.15 |
| Edge column<br>moment about axis parallel to free edge<br>$1.25 + (1.5M_t / V_t x)$ or | 1.40 |
| moment about axis perpendicular to free edge | 1.25 |
| Corner column | 1.25 |

$x$ is the length of the side of the perimeter considered parallel to the axis of bending.

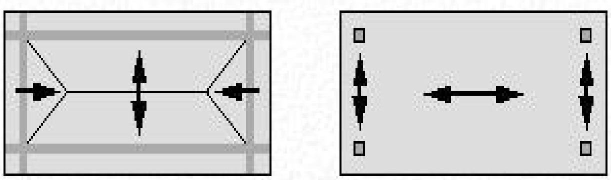
$M_t$ is the moment transmitted from the slab to the column at the connection.

# Deflection



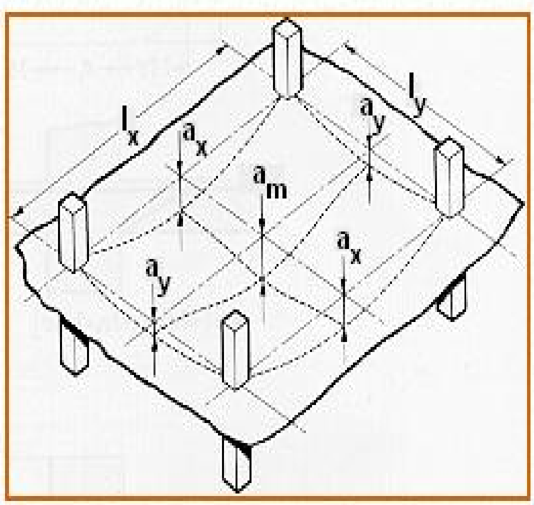
It is self evident that for the same slab stiffness and span, the central deflection of a flat slab is much greater than that of a continuously supported slab.

The two slab systems span thus:



so that as the aspect ratio increases, the flat slab is at a further disadvantage because its deflection increases in proportion to the cube of the longer span, while for the supported slab it is the cube of the shorter span.

A reasonable approximation for calculating the short-term (see Serviceability) mid-span deflection $a_m$, is to summate the mid-span deflections $a_x$ and $a_y$ calculated from an equivalent frame analysis.

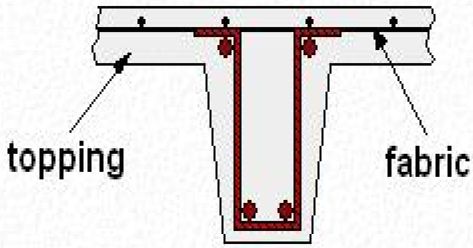



$$a_m = a_x + a_y$$

# Shear Reinforcement

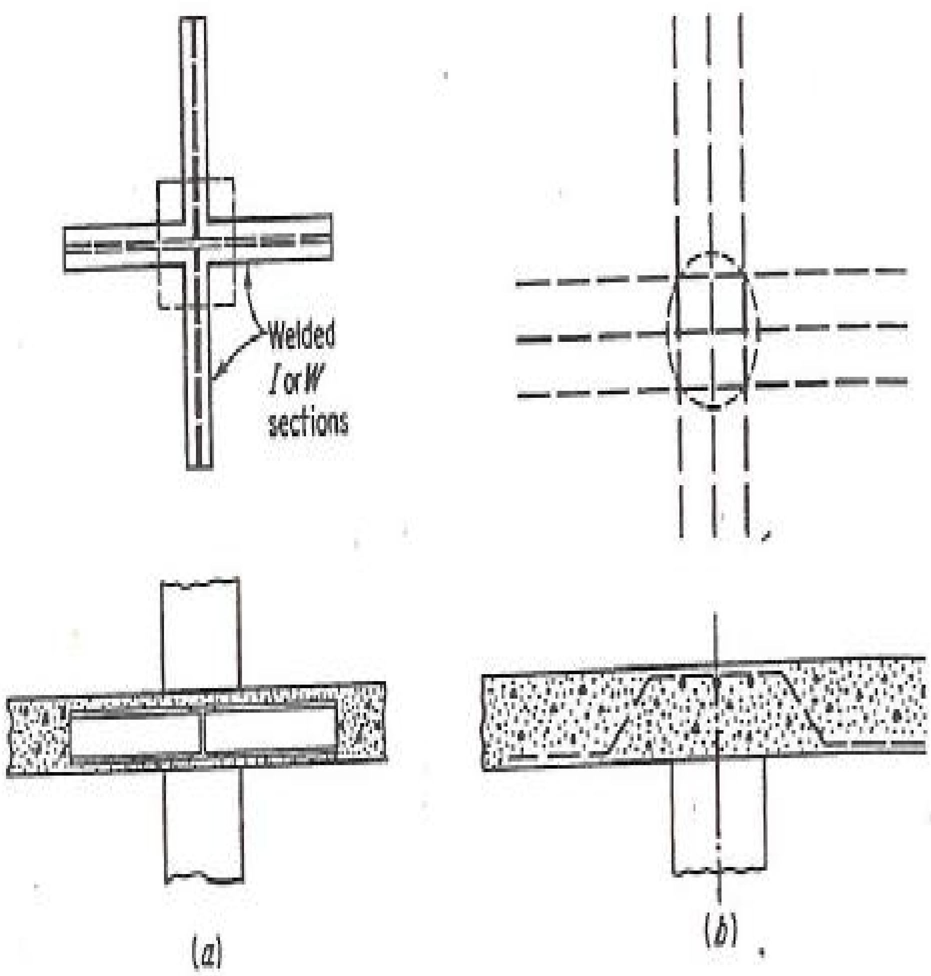
Traditionally shear reinforcement is not provided in solid slabs to resist direct shear forces. If the shear force exceeds the shear resistance of the slab then the design parameters are altered, say by increasing the depth of the slab.

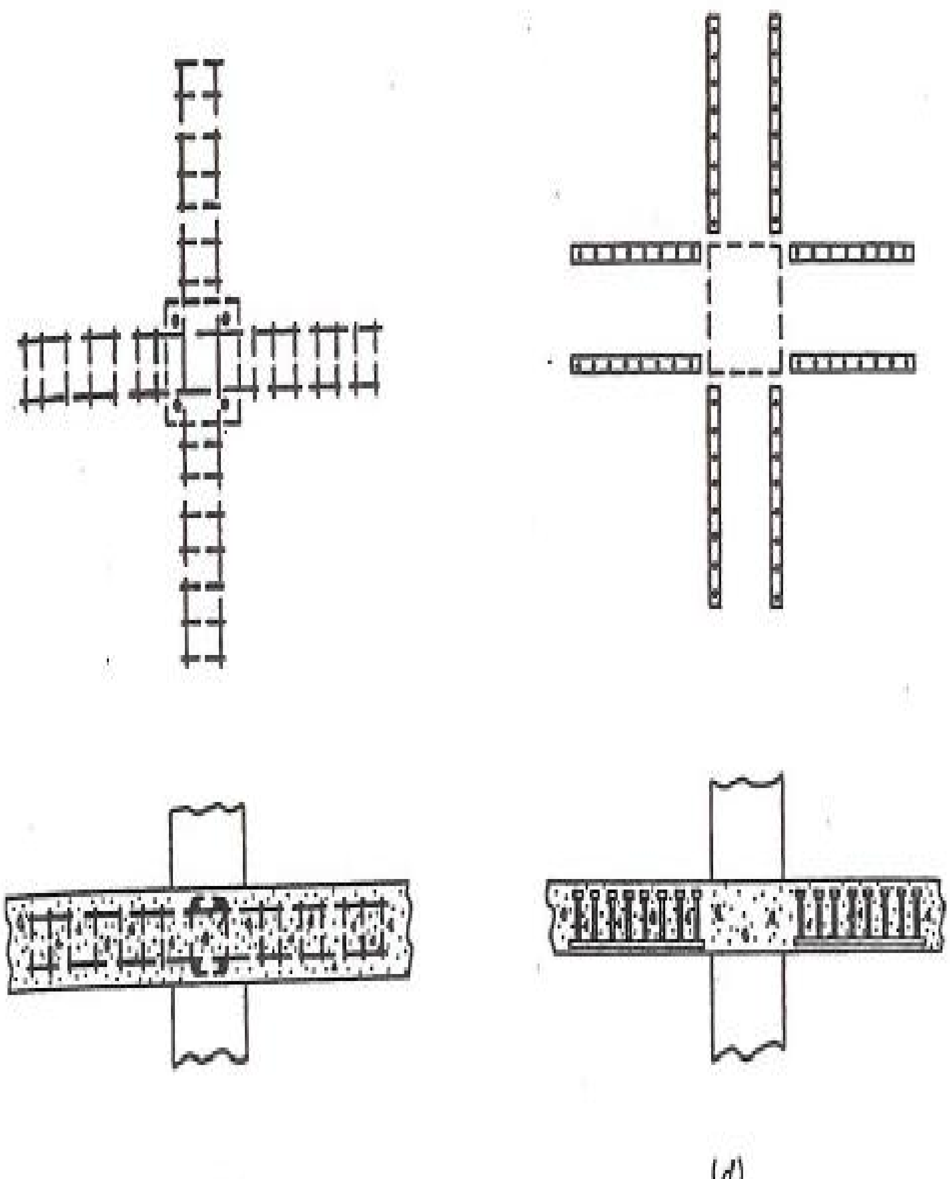
However, this is not the case with ribbed slabs which are designed to carry heavier loads, and where it is normal practice to provide shear reinforcement in each rib. The typical link arrangement below also shows the common practice of providing a nominal percentage of fabric reinforcement in the structural topping.




Shear reinforcement may be required around the supporting columns in flat slab construction to resist the effects of punching shear. This can be in the form of links as shown, but there are alternative solutions which are discussed in the topic 'Punching Shear'.

# Due to the limited depth special reinforcement are required involving the use of shear heads and anchor bars and wires.





(c) (d)

## EXAMPLE 14.1

A floor slab in a building where stability is provided by shear walls in one direction (N–S) and by stairs and lift well in the other direction (E–W) is divided into bays as shown in Fig. 14.7. The slab is to be without drops and is supported internally and on the external long sides by square columns with heads. The imposed loading on the floor is 5.0 kN/m$^2$ and an allowance of 2.5 kN/m$^2$ should be made for finishes, etc. The exposure conditions will be mild and the fire resistance period is half an hour. Use Grade 40 concrete and reinforcement Grade 460, Type 2.

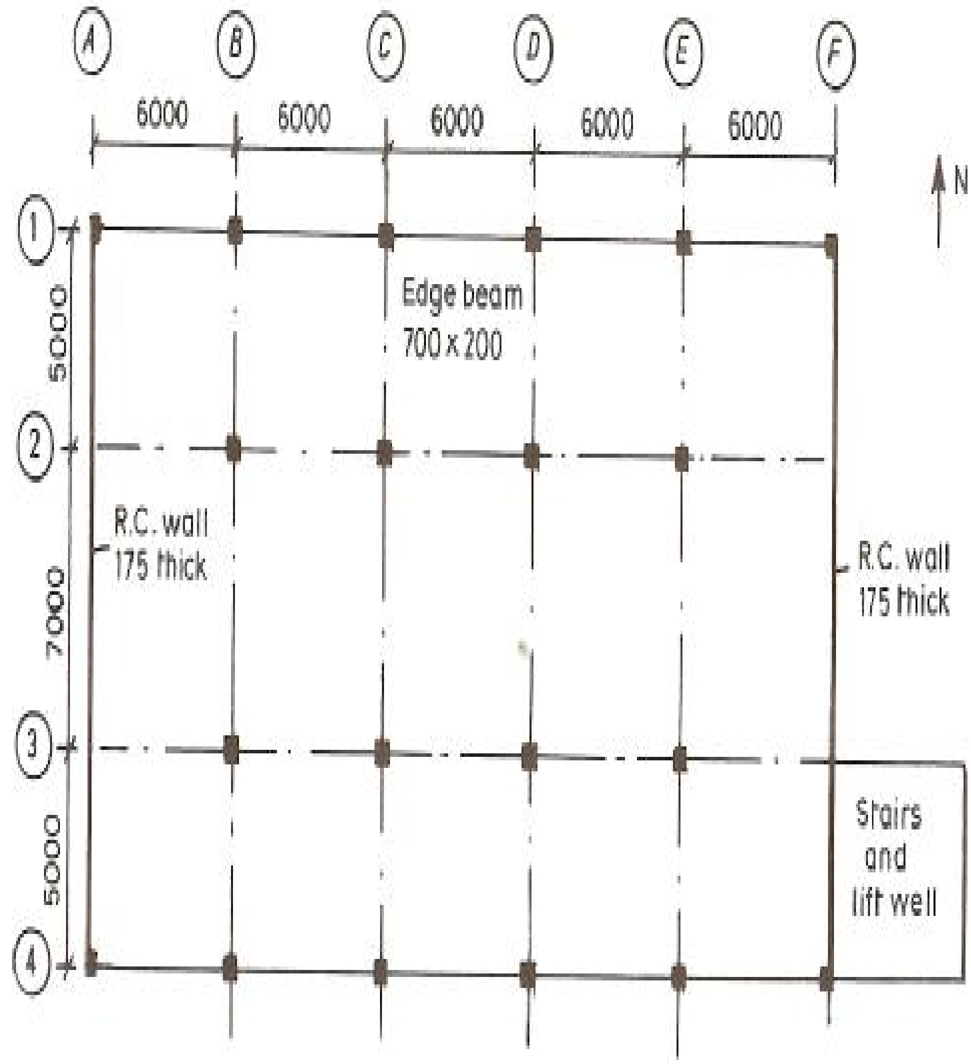


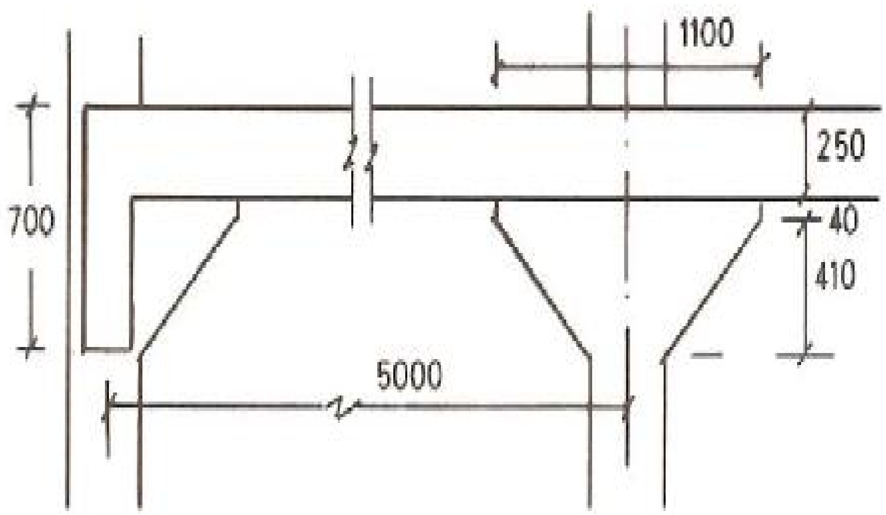

FIG. 14.7 Layout of building.

For the slab thickness assume a span/effective depth ratio of 33. The longest span is 7.0 m so $d$ would be 212 mm. Assuming 20 mm cover and 12 mm bars the overall depth would be 238 mm. Assume a 250 mm slab.

As the columns are square, the heads will also be made square. The maximum value of $h_c$, the effective diameter, is one-quarter of the smallest span, so $h_c = 5.0/4 = 1.25$ m. If the column head is 1.1 m square then $h_c = 1.24$ m. Now decide on the depth of the head.

From clause 3.7.1.3, $l_{hmax} = l_c + 2(d_h - 40)$.
So $1100 = 300 + 2(d_h - 40)$, i.e. $d_h = 440$ mm.

Make head 450 mm deep overall and make edge beam 700 mm deep overall.




Characteristic loads:

Dead, slab $= 6.0\ \text{kN/m}^2$

finishes $= 2.5\ \text{kN/m}^2$

Total $= 8.5\ \text{kN/m}^2$

Imposed $= 5.0\ \text{kN/m}^2$.

Maximum design load $n = 8.5 \times 1.4 + 5 \times 1.6 = 19.9\ \text{kN/m}^2$.

## Analysis

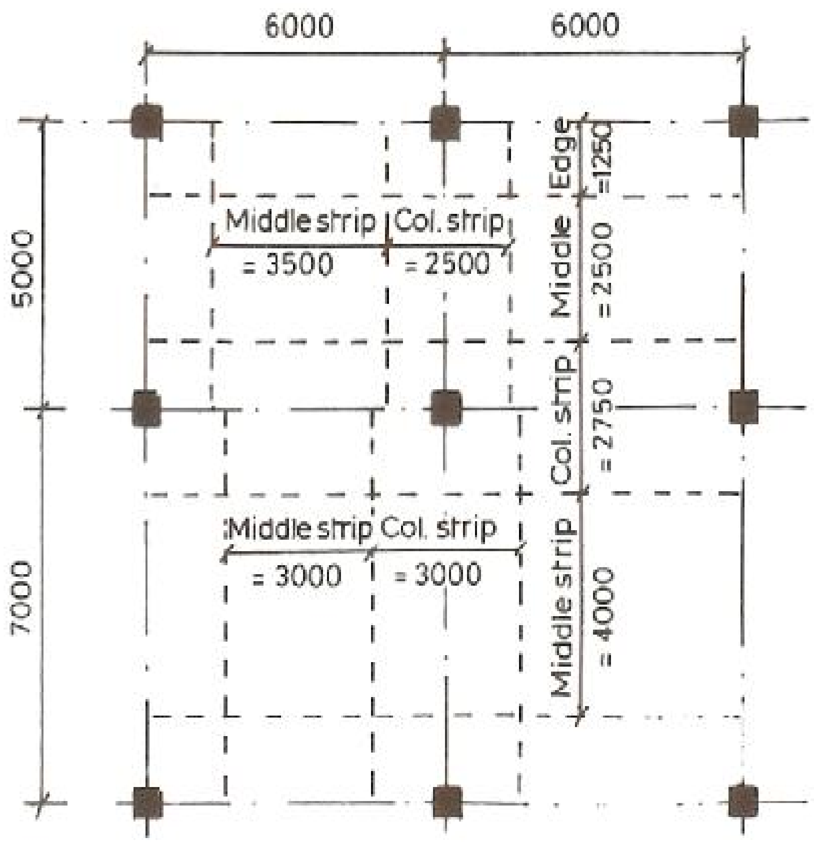
As wind forces in both directions are resisted by shear walls or equivalent the frames in both directions can be classes as braced.

In the north–south direction, as the characteristic imposed load does not exceed 1.25 times the characteristic dead load we can use an equivalent frame analysis considering the single load case of maximum design load on all spans provided that we adjust the support moments and span moments in accordance with the solid slab design requirements given in clause 3.5.2.3.

In the east–west direction, we can analyse as for the north–south direction, but as we have equal spans and more than three rows of panels in the direction being considered we could use the simplified method given in clause 3.7.2.7.

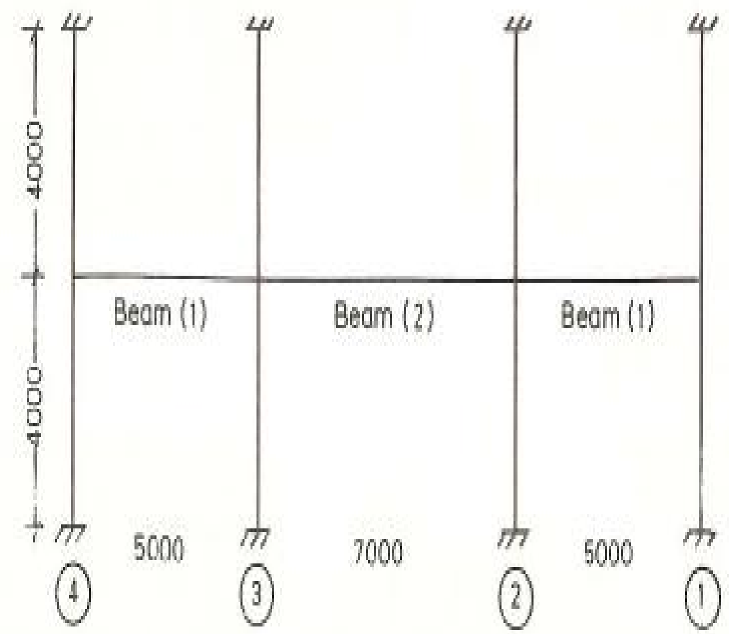
It is proposed to use a subframe analysis in both directions, but a comparison will also be given for the east–west direction using the simplified method.

The panels will be divided into strips as shown below.



## North–south direction

On the line of the internal columns the width of the strip for loading and analysis is 6.0 m, so the maximum design load per unit length = 19.9 × 6 = 119.4 kN/m.




Stiffness:

Columns $I = 300 \times 300^3/12 = 0.675 \times 10^9\ \text{mm}^4$,

$k_c = (0.675 \times 10^9)/(4 \times 10^3) = 0.169 \times 10^6\ \text{mm}^3$.

Equivalent beam:

$I = 6000 \times 250^3/12 = 7.81 \times 10^9\ \text{mm}^4$.

Beam (1)

$k_{b1} = (7.81 \times 10^9)/(5 \times 10^3) = 1.56 \times 10^6\ \text{mm}^3$

Beam (2)

$k_{b2} = (7.81 \times 10^9)/(7 \times 10^3) = 1.116 \times 10^6\ \text{mm}^3$.

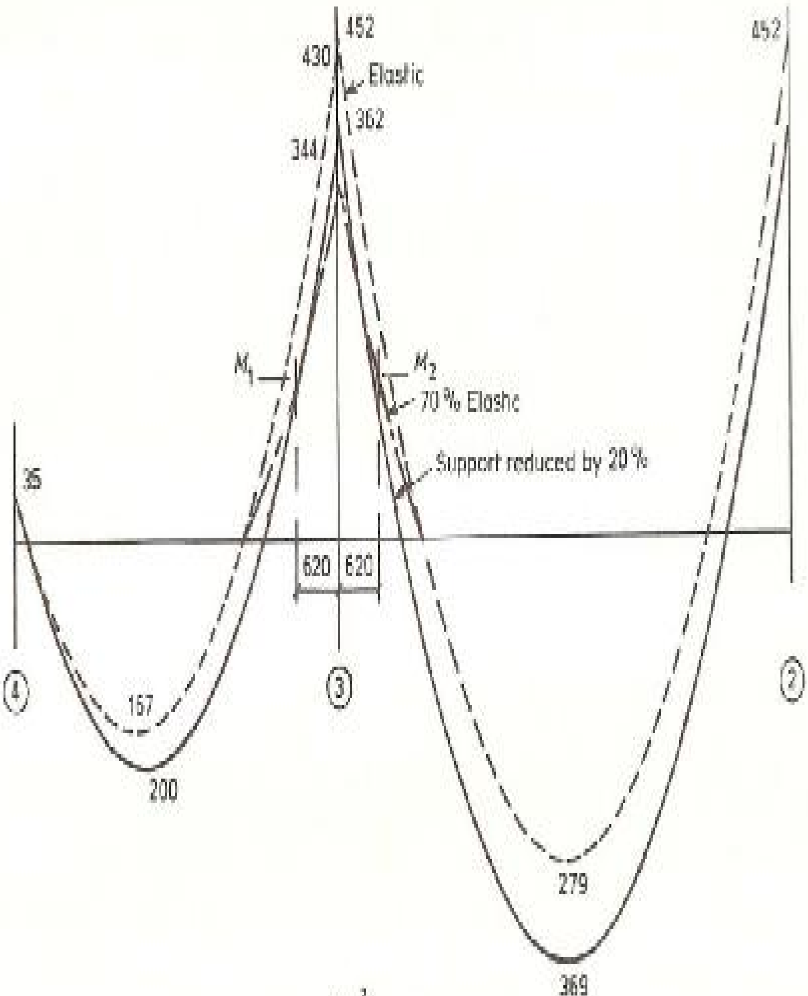
From an analysis of the single load case of all spans loaded with the maximum design load we get the following results:

| 4 | 3 | 2 | 1 | | |
|---|---|---|---|---|---|
| −35 −430 | −452 −452 | −430 −35 | | Support | Beam moments |
| +167 | +279 | +167 | | Span | Beam moments |
| 17.5<br>17.5 | 11<br>11 | 11 17.5<br>11 17.5 | | Upper<br>Lower | Column moments |
| 220 377 | 418 418 | 377 220 | | Shear | |

The bending moment diagram is shown in Fig. 14.8.

We can now reduce the support moments by 20%, but as the exterior support is not very large, we will reduce the internal support only. The span moments will increase accordingly. We also have to comply with the 70% maximum moment requirement.

N




$$M_1 = 0.7\left(-430 + 377 \times 0.62 - 119.4 \times \frac{0.62^2}{2}\right) = -153$$

$$M_2 = 0.7\left(-452 + 418 \times 0.62 - 119.4 \times \frac{0.62^2}{2}\right) = -151$$

FIG. 14.8 Bending moment diagram.

From the bending moment diagram it can be seen that this 70% line controls part of the new diagram. We are now allowed to reduce the support moments even further by taking the moment at $h_c/2$ from the centre line of the column, provided the sum of the positive moment and average of the negative moments is not less than $19.9 \times \frac{6}{8}(l - 2 \times 1.24/3)^2$ where $l$ is 5 m and 7 m respectively for the short and long span. So short-span value = 260 kN m and long-span value = 569 kN m. In the short span at the external column the moment will be positive if we take the value at $h_c/2$ from the centre line of the support. This will be ignored and the value of 35 will be used. The average of the negative moments is therefore $(35 + 153)/2 = 94$, which added to the span moment of 200 gives a total of 294; this is in excess of the required moment.

In the long span the sum of the moments is $151+369=520$, which is less than the minimum value. The negative moments must be increased by 49 to 200 to satisfy the minimum requirement.

For the external column the maximum moment which can be transferred is given by

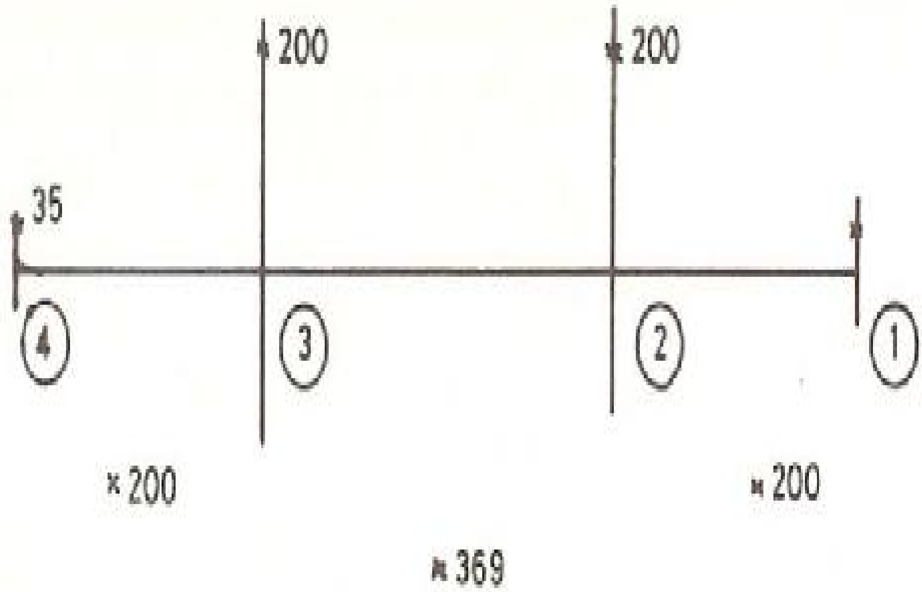
$$M_{t,max}=0.15b_e d^2 f_{cu}$$
$$b_e=300+250=550$$
$d=250-20-8=222$ mm (assuming $16\phi$ bars which will be in outer layer).

So

$$M_{t,max}=0.15\times550\times222^2\times40\times10^{-6}=163 \text{ kN m}$$

which is well in excess of actual moment.

The design moments can be shown diagrammatically as follows:




At the first interior support, i.e. at the junction of the 5 m and 7 m spans, we have the problem that the column strip in the 5 m span is 2.5 m while in the 7 m span it is 3 m, so the wider column strip will be used.

The moments from analysis can now be apportioned in accordance with Table 3.20 of the Code.

## Column strip

Exterior support

$$= 0.75 \times 35 = 26.3 \text{ kN m on strip 2.5 m wide} = 10.5 \text{ kN m/m.}$$

Centre of first span

$$= 0.55 \times 200 = 110 \text{ kN m on strip 2.5 m wide} = 44.0 \text{ kN m/m.}$$

First interior support

$$= 0.75 \times 200 = 150 \text{ kN m on strip 3.0 m wide} = 50.0 \text{ kN m/m.}$$

Centre of interior span

$$= 0.55 \times 369 = 203 \text{ kN m on strip 3.0 m wide} = 67.7 \text{ kN m/m.}$$

## Middle strip

Exterior support

$$= 0.25 \times 35 = 8.8 \text{ kN m on strip } 2.5 \text{ m wide} = 3.5 \text{ kN m/m.}$$

Centre of first span

$$= 0.45 \times 200 = 90 \text{ kN m on strip } 2.5 \text{ m wide} = 36.0 \text{ kN m/m.}$$

First interior support

$$= 0.25 \times 200 = 50 \text{ kN m on strip } 3.0 \text{ m wide} = 16.7 \text{ kN m/m.}$$

Centre of interior span

$$= 0.45 \times 369 = 166 \text{ kN m on strip } 3.0 \text{ m wide} = 55.4 \text{ kN m/m.}$$

For positioning the reinforcement, the bars in this direction will be in the outer layers, so assuming $12\phi$ bars, $d = 250 - 20 - 6 = 224$ mm.

Minimum area $= (0.13/100) \times 1000 \times 250 = 325 \text{ mm}^2/\text{m}$.

Use $12\phi$ at 300 centres (377 $\text{mm}^2$/m).

With this area of reinforcement the moment of resistance can be calculated as 32.1 kN m/m. Any moment less than this value will be covered by the minimum reinforcement. It can also be found that $12\phi$ bars will be satisfactory in all positions.

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## Deflection – clause 3.7.8

)N Consider interior span. In the span the total moment is 369 kN m on a band 6.0 m wide, so $M/bd^2 = 1.23$.

If we assume that the reinforcement provided is exactly the amount required, Table 3.11 of the Code gives a modification factor for tension reinforcement as 1.30.

The allowable $l/d = 26 \times 1.3 \times 0.9 = 30.4$ which gives an allowable span of 6.8 m. This is not quite sufficient, but by providing more reinforcement than is required it will be found to be satisfactory.

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## Shear

Internal column using equation (25) of the Code:

$V_t = 795$ kN

$d = (224 + 212)/2 = 218$ average

$x = 1100 + 3 \times 218 = 1754$ mm

$u = 4(1100 + 3 \times 218) = 7016$ mm

$M_t = 11 + 11 = 22$ kN m

$$V_{eff} = 795\left(1 + \frac{1.5 \times 22 \times 10^6}{795 \times 10^3 \times 1754}\right)$$

$$= 795 \times 1.024 = 814 \text{ kN.}$$

Note that the enhancement factor of 1.15 as suggested in the Code appears to be conservative in this case.

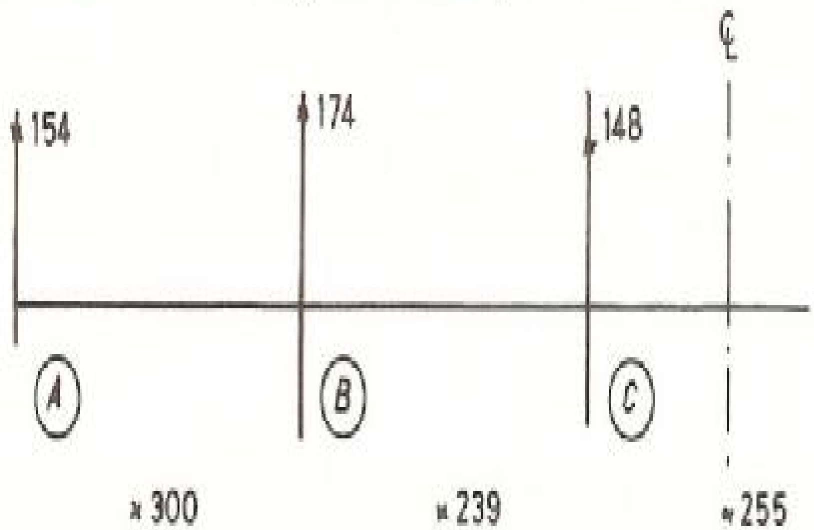
$v = (814 \times 10^3)/(7016 \times 218) = 0.53$ N/mm$^2$.

This cannot be checked completely until the reinforcement in the direction at right angles has been calculated, but working the reverse way from Table 3.9 of the Code we shall need an average percentage of reinforcement of approximately 0.25%. At the column head itself it can be found that $V_{eff} = 825$ kN and $v_{max} = 0.86$ N/mm$^2$.

## East–west direction: frame analysis

The strip for loading and for analysis will be 6.0 m wide. The exterior support is a continuous wall 175 mm thick, and we shall take a 6.0 m length, the same as the beam strip.

The maximum design load per metre run will be the same as in the north–south direction. By carrying out the same procedures as before we can arrive at design moments which can be shown diagrammatically as follows:



From the layout of the division of panels it can be seen that the column strip is 2.75 m wide, and the middle strip is $6.0 - 2.75 = 3.25$ m, which is made up of 2.0 m from 7 m wide panel and 1.25 m from 5 m wide panel.

The moments in the strips would now be apportioned in accordance with Table 3.20 of the Code, and in calculating the areas of reinforcement it must be remembered that $d$ will now be 212 mm. The minimum reinforcement will be as before, but due to the slightly reduced effective depth will provide a smaller moment of resistance.

The deflection will be checked on the exterior span. As the reinforcement in both directions is now known the shear around the column heads can also be checked.

## Simplified method

Maximum design load per unit length $= 19.9 \times 6 = 119.4$ kN/m.
From Table 3.19 of the Code, $F = 119.4 \times 6 = 716.4$ kN.

Effective span, $l = 6 - 2 \times 1.24/3 = 5.17$ m for internal panels

and

$l = 6 - (1.24 + 0.175)/3 = 5.53$ m for external panels.

Bending moments:

Outer support (classed as column) $= -0.04 \times 716.4 \times 5.53$
$= -158.5$ kN m.

Near centre of first span $= 0.083 \times 716.4 \times 5.53$
$= 329$ kN m.

First interior support $= -0.063 \times 716.4 \times 5.53$
$= -250$ kN m
or $-0.063 \times 716.4 \times 5.17$
$= -233$ kN m
(The larger value would be used).

Centre of interior span $= 0.071 \times 716.4 \times 5.17$
$= 263$ kN m.

Interior support $= -0.055 \times 716.4 \times 5.17$
$= -204$ kN m.

If one compares these moments with those from the frame analysis it can be seen that the simplified method gives higher values in all cases. These are particularly noticeable at the interior supports; this is because the simplified method does not make allowance for the flared column head.

For shear we obtain

| | Frame analysis | Simplified |
|---|---|---|
| (i) Exterior support | 322 kN | 322 kN |
| (ii) First interior support | 760 kN | 788 kN |
| (iii) Interior support | 708 kN | 716 kN |

which are very similar.

The simplified method, however, will give considerably higher column moments.

## 14.10 Arrangement of reinforcement

Where a frame analysis has been carried out and a bending moment envelope derived, the reinforcement will be curtailed in accordance with the normal detailing rules of clause 3.12.9. For the simplified method of analysis the simplified rules of clause 3.12.10 may be used.

Whichever method is used, the column strip reinforcement over the column head should be arranged so that two-thirds of the amount of reinforcement required should be placed in a width equal to half that of the column strip and central with the column.

The detailing for the simplified method for slabs as shown in Figure 3.25 of the Code gives dimensions from the face of the support as a proportion of the effective span. For columns with heads the face of the support will be taken as the edge of the head. As these heads can be classed as wide supports, the effective span will be the clear distance between heads plus the effective depth of the slab. The dimensions for the middle strip reinforcement will be the same as for the column strip.